Math 126 Summer 2026 HW 1

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Problem 1: (Borthwick 2.2) Polar coordinates in $\mathbb{R}^2$ are related to polar coordinates by $(x^1, x^2) = (r\cos(\theta), r\sin(\theta))$.

A.) Use the chain rule to compute $\frac{\partial}{\partial r}$ and $\frac{\partial}{\partial \theta}$ in terms of $\frac{\partial}{\partial x^1}$ and $\frac{\partial}{\partial x^2}$.

B.) Find the expression for Δ in radial coordinates.

Problem 2: Let $E = \{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{16} + \frac{y^2}{9} \leq 1\}$. A.) Describe the boundary of the set. Is this set open or closed?

B.) What is the outward unit normal to the boundary at the point $(2\sqrt{2}, \frac{3}{\sqrt{2}})$?

Problem 3: (Borthwick 2.4) For $f \in C^0(\mathbb{R}^n)$, set $h(t) = \int_{B(0,t)} f(x)dx$. What is $\frac{dh}{dt}$? (Hint: use the radial decomposition to break the volume integral into an integral over the radius and over the sphere)

Problem 4: (Borthwick 3.1) Consider a conservation equation with constant velicoty $c > 0$,

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$$

in the quadrant $t, x \geq 0$. Suppose that $u(0, x) = g(x)$ for $x \geq 0$ and $u(t, 0) = h(t)$ for $t \geq 0$, where $g, h \in C^1([0, \infty))$.

A.) Find a formula for $u(t, x)$ in terms of g and h.

B.) What condition on g,h do we need for u to be continuous?

Problem 5: Let $U \subset \mathbb{R}^2$ be the half-space $\{x^2 > 0\}$. Consider the partial differential equation

$$\begin{cases} u_{x^1} + u_{x^2} = u^2 & U \\ u|_{x^2=0} = (x^1)^2 \end{cases}$$

A.) Compute the characteristic ODEs.

B.) Solve for $x(s)$ and $z(s)$ from the characteristic ODEs.

C.) Solve for $u(x^1, x^2)$ as a formula depending on x^1 and x^2.

D.) Check that u is truly a solution to the ODE.